Concurrent Time

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(Dated: May 26, 2009)

Abstract. This essay aims at introducing a novel point of view on the nature of time, inspired by a synthesis of three seemingly unrelated concepts: Bergson's notion of duration, Dijkstra's notion of concurrency, and Mach's notion of inertia.

"(...) science cannot deal with time and motion except on condition of first eliminating the essential and qualitative element – of time, duration, and of motion, mobility."

Henri Bergson (Time and Free Will)

"(...) the first challenge for computing science is to discover how to maintain order in a finite, but very large, discrete universe that is intricately intertwined."

Edsger W. Dijkstra (My Hopes of Computing Science)

"[The] investigator must feel the need of... knowledge of the immediate connections, say, of the masses of the universe. There will hover before him as an ideal insight into the principles of the whole matter, from which accelerated and inertial motions will result in the same way."

Ernst Mach (The Science of Mechanics)

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I. INTRODUCTION

Time is the most "undefinable definable" thing.

It is "definable" because it enjoys an operational existence as a number. It is "undefinable" because it eludes mathematics. Both these statements cannot enjoy a true condition together. The frontier that divides these irreconcilable territories is nebulous, and this is where we stand. We recognize the paradox, and its resolution lies in sharpening the fuzzy frontier that will conciliate both worlds. One may live forever in the former land of "definability" as far as one's routinely necessities are fullfiled. But it is undeniable that mankind has also stepped into the fuzzy frontier beyond the utilitarian land, into the "undefinable". We have realized the puzzle of our own existence. The dilemma of the co-existence of both worlds now stands before our eyes.

Physics is mainly an utilitarian discipline (although of conceptually far reaching consequences) that relies on mathematics for the formal structure of its ideas and descriptions, and hence, time can be made "definable". From such an utilitarian discipline, we scratch the surface of nature. In general relativity, time is a coordinate, like its spatial counterparts. In quantum mechanics, time is an external parameter. These theories are examples of our most successful fundamental descriptions of nature, but have different domains of validity and use distinct formalisms. These theories are presently irreconcilable, but no matter how different they are, they ultimately express any succession of different configurations of a system as a linear counting of coincidences[1]. Therefore, despite their conflicting views of how to express such a counting − which is actually part of the problem in making them conciliable −, these theories actually share a common operational tool for dealing with time. They adopt numbers to delineate dynamics − ultimately, an orderly sequence of events. *The cold immobility of numbers prevails.*

As the french philosopher, Henri Bergson, so keenly expressed it ([3], page 234):

> But if time, as immediate consciousness perceives it, were, like space, a homogeneous medium, science would be able to deal with it, as it can with space. Now we have tried to prove that duration, as duration, and motion, as motion, elude the grasp of mathematics:

[1] One may be able to construct a generalized non-relativistic quantum mechanics for closed systems in which the notion of a "state at a moment of time", evolving unitarily, is no longer introduced; see, e.g., [14].

> of time everything slips through its fingers but simultaneity, and of movement everything but immobility. (...) [I]n this so-called phenomenal world, which, we are told, is a world cut out for scientific knowledge, all the relations which cannot be translated into simultaneity, i.e. into space, are scientifically unknowable.

Bergson refers to the "undefinable" time, which has been "cut out for scientific knowledge", that is, cut out into the "definable" time. On the other hand, a complex system like the brain, a result of a long series of evolutionary irrevesible incidents, "creates time" for itself – the "undefinable" time, that which Bergson is at home, in pure intuition. Hereon, the word "time" alone will refer to that nebulous frontier between the "definability" and "undefinability" lands – that frontier which we attempt to sharpen, even though in a minuscule way. This fuzzy border – time – is the object of our interest and the purpose of this essay. It is something fundamentally quite different from our utilitarian "immobile numbers" at one side of the territory, and from the pure Bergsonian characterization of it, at the other side. It demands some novel ontological structural characterization, yet to be conceived. Whether it can be elevated to a physically testable hypothesis is evidently an important question. The easy answer lies on how precise and intimate the relation between the working of the brain and irreversibility in complex systems (including quantum systems) is, and how such a relation can be fundamentally addressed as scientific knowledge progresses. The hard answer is... unknown, and perhaps an impossibility.

We will be guided by three apparently disparate ideas: starting from Bergson's duration as the basis of our journey, we set the bridge from it to the notion of "concurrency" as pioneered by Dijkstra[2] , and then focus our view of the brigde from a high altitude – the "inertia" – inspired by Mach, which was partially advanced by Einstein in his general relativity theory. These various elements, constructions and view sites will be approached under a hopefully new light. One may ask – what are the motivations in looking for guidance on so much disparate thinkers? The answer is that those schools of thought do have a very common attribute: *they hide an underlying vision of time in which the cold immobility of numbers has no place*, and hence, observe from a distance the fuzzy frontier under different, perhaps privilleged positions.

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[2] An online repository of his works can be accessed in [8].

II. DURATION IS CONCURRENCY: INTUITION TOWARDS THE HETEROGENEOUS TIME

Bergson was a remarkable "inner" traveller and wonderful writer. He had a unique intuitionist approach to difficult philosophical questions. His concept of "duration" offers a window to respond the question at hand: how to perceive time in one's most intimate self, independently from anything else exterior to the mind? It is a snapshot to the brain's treatment of irreversibility – coming from inside the brain itself. Reversible laws are based on a tacit assumption of an equivalence between time and spatial simultaneities, namely, a reduction to a properly defined orderly sequence of events expressed, in turn, by numbers. From simple constitutive laws to many-body interactions, lies an unescapable ignorance index, cleverly explored under statistical treatments, transporting the "definable" time all the way through. The process often results in a powerful tool for describing general complex behaviour in idealized conditions. However, although it can be casted into a well-understood problem (under formal aspects), there is no clear agreement on how exactly the emergence of irreversibility can be deduced from the fundamental reversible laws, specially under certain physical conditions (see, e.g., [6, 13, 16, 19, 20, 23]). Such a difficulty is seen by some as a technical one, by others as a fundamental one. In the present essay, we adopt the latter point of view.

Bergson's definition of his celebrated "duration" is not very straightforward, specially because he develops an elaborate theory of time intrinsically connected with consciousness, which also attempts to clarify the problem of free-will as a criticism to Kantian philosophy. So it is a complex analysis. It is not the purpose of this essay to present an erudit exposition of it. The reader is urged to read Bergson's work directly from his books[3]. *Duration* is basically introduced as *heterogeneous time*. According to the philosopher, our main error is an "illegitimate translation of the unextended into the extended", or a "confusion of duration with extensity, of succession with simultaneity, of quality with quantity" ([3], preface). Recognizing the difficulty of the concept, Bergson offers many visual aids or examples in order to illustrate it (see a summary, e.g., in Ref. [24]). "Duration" is a qualitative multiplicity – as opposed to a quantitative multiplic-

[3] The writting of this essay was inspired on a study of two his well-known masterpieces [3] and [4].

ity that we project into space as things that we follow as a sequence of snapshots, only differing by their constitutive relations translated into simultaneities. According to the Stanford Encyclopedia of Philosophy[24]:

> A qualitative multiplicity is therefore heterogeneous (or singularized), continuous (or interpenetrating), oppositional (or dualistic) at the extremes, and progressive (or temporal, an irreversible flow, which is not given all at once). Because a qualitative multiplicity is heterogeneous and yet interpenetrating, it cannot be adequately represented by a symbol; indeed, for Bergson, a qualitative multiplicity is inexpressible.

What we see "outside" ourselves is pure quantitative multiplicity − its changes, identified as "the passage of time" into an orderly sequence of events is just a rudimentary, poor transcription of an underlying "time". Such a deficiency or reduction of the underlying perception of time would be a result of natural selection mechanisms, which delineated the way that our biological entities perceive the material world under what is relevant for immediate survival. According to Bergson[4]:

> The human intellect feels at home among inanimate objects, more especially among solids, where our action finds its fulcrum and our industry its tools; that our concepts have been formed on the model of solids; that our logic is, pre-eminently, the logic of solids; that, consequently, our intellect triumphs in geometry (...).

Therefore, the ontological status of time in Bergson's philosophy is structured as a polarization between the internally perceived states of the mind, which are of qualitive and heterogeneous nature, and its externalization as a non-interpenetrating, homogeneous "space" (not to be understood as a real homogeneous space in the physical sense, but in an ontological sense) in which our "logic of the solids"[4] is our natural representation. The externalization of Bergsonian time is the "definable" time; its internalization, the "undefinable" time. By themselves, these dualities appear quite abstract and indeed they are[5]. However, we would like to keep them at perspective and retain their main "spirit" in the arguments that follow.

[4] "Solids" are considered here in a broader context − e.g., the concept of a "field" also fits in this scheme, as it is structured as an intensity as a function of the coordinates − it is a "spatialized" thing, a "solid", in this sense.

[5] It is evident that nothing is gained from intuitions (specially considering that Bergson does put weight on them in order to develop his concept of "duration") if we cannot decide which of our intuitions with respect to the notion of time should be retained and which should be understood as artifacts inherited from our predecessors. This is an unresolved problem at the present point. As the understanding of

In physics, unifying laws and concepts are powerful tools for constructing a big picture for describing phenomena in their generality. But these are "homogoneous" (in a Bergsonian sense) descriptions of nature. What arguments could oppose the possibility that nature's fundamental structure (namely, those ultimately modeled as field properties, spacetime fabric, and other utilitarian concepts) is actually of a "heterogeneous" quality? We here advance the idea that our current notions of time are only a minimalistic facet of a more "heterogeneous"-like nature of an underlying *concurrent substrate*, guided by spontaneous correlations. These correlations, however, are not field or spacetime correlations in the current physical sense, but more fundamental ones. Our interest in those ideas lies from recognizing that the non-interpenetrating and heterogeneous status of Bergson's "duration" can be casted into a qualitative multiplicity *substrate of a concurrent nature*. A bridge over the nebulous frontier between the "duration" and the "immobile sequence of numbers" can be conceptually constructed in which the fundamental blocks are built from an interpenetrating (*not* in physical spacetime sense), *relational set of concurrent, fundamental processes*. Before proceeding, it is important to clarify that Wolfram's ideas [25] address a completely different question than the one posed here. We are not considering the aspects resulting from the consideration of physical laws as computations (they could!), but the assumption that nature embraces a substrate of concurrent elementary processes, from which the quantum realm emerges, leading to the classical world *from the same basic constituents*. But what are these?

III. CONCURRENCY IS INERTIA: THE PROCESS-BASED WORLD TOWARDS NON-LOCALITY

Dijkstra's pioneering view on the mechanisms of concurrent processes are well recognised. He introduced the − now famous − "dining philosophers problem"[6] in 1971

the human brain evolves, and new techniques are available, "intuicionism" (a self-perception of the brain) could eventually become prone to translation and reduction into a qualitative map which could serve as a guide to more quantitative investigations, specially concerning how the various states of the mind imprint the internal perception of time.

[6] Actually, the re-formulation of the problem into a scene of "dining philosophers" is atributed to the computer scientist Sir Charles Antony Richard Hoare (see [11]). This problem has been discussed formally and in more generalized conditions in the context of proofs of correctness in concurrency

[10]: a simple scenario which captures the fundamental problems of concurrency theory, specially, the occurence of *deadlocks*. We have no space to review the general concepts of computational concurrency (we refer the reader to ref. [5] for a starting point). We only touch on a few concepts, which form a useful parallel to the ideas of this essay. A concurrent system can be defined as a collection of two or more simultaneous tasks that dynamically depend on each other in order to fulfill their individual objectives. In other words, concurrent processes, although executing relatively independently, are acting together towards some integrated result. Hence they need to establish some communication and coordination. For instance, processes can read and write to the same memory location one at a time. Dijkstra's "dining philosophers" scenario makes it clear that a multiprocessing system, in which processes in a closed chain share limited, mutually exclusive resources, can eventually enter a state in which no progress is possible, the deadlock state. This is a term that refers to a state of the system in which processes are blocked and waiting for a condition that will never happen.

But concurrency is not only a matter of computer science. It is ubiquitous and obvious in nature to the point that we simply ignore it. This is true except in engineering and computational design problems, where the concurrent behaviour of the external world, including how information is processed and transmitted, must be worked out and anticipated quite explicitly. Interestingly, it has not always been that way. The more "traditional" sequential modeling started first, but it is just a special case of concurrent modeling, and it is much easier to implement[7]. But as computational facilities evolved, a shift of paradigm from sequential to concurrent modeling started to gain importance, and programmers started to develop techniques to deal with concurrency. But why do we take concurrency so much for granted in physics problems? Indeed, the evolution of a given set of independent physical elements (for instance, spacelike events, uncoupled fields, etc) can be argued to be at most an uninteresting case of concurrency. But when correlations arise, additional terms are imposed to describe the coupling of fields, lightlike events are causally connected, nonvanishing commutation relations arise, etc. But even when physical elements interact with each other − which is the norm in our Universe

programming (e.g., [15]).

[7] Note that mathematical equations expressing physical laws are also "sequential" in the sense of an "input-calculation-output" machine.

−, concurrency can be thought of, at best, as an acceptable *a posteriori* description, although perhaps dispensable or redundant.

That is not the way we think the issue should be addressed. Physics is concerned with dynamical laws, supposed to describe how a given configuration of self-interacting constituents evolves, given a "time span", to another configuration. In such a framework, any concurrency-like characterization would be taken as deducible from the dynamics, and never the other way around. *But what if we revert that logic and consider that nature is concurrent in a fundamental sense?* If concurrency is some deeply inherent property of the world, then it would not be a consequence of dynamics, but the fundamental cause of it. If this hypothesis stands correct, then an insight should be gained on the nature of time, by making our theories *explicilty concurrent.* Concurrency should be elevated as a new internal symmetry of the world, in which the probabilistic framework of quantum mechanics would be just an emergent and incomplete facet of a more heterogeneous underlying substrate. And the reversible or irreversible behavior of a given system must somehow be attributable to the underlying heterogeneous concurrent correlations.

Let us take, for example, a brief consideration on quantum mechanics. Quantum theory is a local theory in terms of the interactions occurring between any two systems, but the description of the composed system in terms of its quantum states is a completely nonlocal one, because it is necessary to include the entangled states of the composite system in order to account for the quantum correlations between the two systems. This gives rise to quantum behavior that is quite contrary to classical, common sense expectations. However, it is exactly the delocalization of the state corresponding to the physical system "under observation" and the environment, the entangled system-environment state, given by the phenomenon of "decoherence", which effectively must explain the emergence of the classical world from the underlying quantum one. Under our hypothesis, the concurrent substrate should be the lowest level at which entanglement operates, and it should be also the lowest level at which "decoherence" takes its form. In a concurrent language[8], we would say that quantum interacting systems are concurent systems in which internal processes share a "common resource" (it would correspond

[8] Note that the fundamental "concurrency" notion that we address here is not the pure-state entanglement measure, the so-called "concurrence"[21], which is a mere quantification resource, although a formal relation between the two could in principle be attempted.

to a mediating fundamental field or propagator in an emergent sense). The quantum state is the result of an internal, non-local (i.e., with no reference to spacetime locations) concurrent substrate.

It is a most interesting fact that correlations of physical properties between two entangled particles, as so far tested experimentally, cannot be explained under local realism. It would be a fundamental step to examine how to restore realism under the concurrent hypothesis here addressed (realism not towards local spacetime locations, but towards a lower level of reference). Time would be the result of the concurrency of the underlying operations of competing "processes", ever present and ever existing, of a qualitative and heterogeneous nature, resulting in an irreversible operation on the next level of realization, in which spacetime localization is realized.

However, *if the Universe is a finite, huge concurrent system, it is prone to deadlocks*. What would be a physical realization of a deadlock situation?

That is a quite dramatic consequence to our fundamental concurrency hypothesis. Let us appreciate it by using a concrete model, such as the following. In a n-process concurrent system there can exist "forbidden regions" or "topological holes" represented in a n-cube space (spanned by trace processes which perform actions on the system) which are inaccessible to the concurrent processes due to mutual exclusion of their shared resources. A geometric view of the situation is the well-known "progress graph" [12], which can be generalized to "ditopological spaces" (topological spaces with a local partial order, see, e.g., [9]). "Unsafe regions" would indicate a route towards an inescapable deadlock situation. Nature's own existence would be dictated by the spontaneous and interpenetrating "topology" of a countably infinite number of holes in a ditopological space. But what would physically correspond to a deadlock state? In order to address these questions, we have to consider mobility, and hence, inertia.

IV. INERTIA IS TIME: MOBILITY TOWARDS THE WHOLE

Mach had a relational view of mechanics that touched upon on of the most fundamental elements of motion: how does inertia arise? Contrary to what many believe, the origin of inertia is still under debate [2]. Mach's comment on Newton's bucket experiment imposes an inescapable holistic view in which local inertial reference frames are somehow determined from the universe at large. Mach's idea played a fundamental role in the developments of general relativity (GR), but at what level does GR implement it, is a question of debate. The question raises confusing issues since there are in fact several interpretations for this principle in the context of different classical theories of gravitation (see the index on page 530 of Ref. [2]).

Mach's principle in the present essay is regarded as a general philosophical questioning concerning mobility. Is inertia an intrinsic quality of material bodies? If one removes the rest of the universe, does a lonely particle have inertia? Does time pass to this lonely particle? Does inertia exist in the sub-Planckian realm? We believe that the question of the origin of inertia is a genuine unsolved problem, which can only be fully understood from a holistic synthesis of how dynamics is continuously embraced in the Universe at all scales. In our point of view, this can only be possible by assuming a unifying framework at a deeper level, which operates independently of the emerging structures which we call spacetime, field intensities, etc.

In order to have a more concrete example of what we have in mind, we consider quantum gravity for a moment (even though there is still no accepted, complete theory of it!). We consider the possibility that the notion of a spacetime event (point) breaks down nearby the Planck scale. We would like to replace the notion of a point with the notion of a "smeared" point, or more precisely, an open set of concurrent processes. The spacetime foam would be represented by a coherent superposition of sets of processes that act concurrently, and the reason that they act concurrently lies in the fact that they superpose each other at certain extent. (Note that such a superposition is not in space, because the processes themselves should represent a relational framework in which spacetime emerges at the classical level). *It is a superposition constrained by causality*. Causality should somehow represent the local "shared resource" among the coherent set of processes. This would be the reason why they act concurrently: they are *causally constrained* by countably infinite abstract shared resources. A quantum state of a volume spacetime could only evolve as the result of the combined act of the fundamental processes that compete for a common resource: causality itself.

As a very simple model of "concurrent inertia", each set of concurrent processes could be represented by a ditopological space, and each ditopological space would be partially mapped into some other because of the local causality constraint. The ditopological spaces and their common (causal) superpositions maps (the abstract shared resources) should have a combined effect of correlations and anti-correlations among processes (that is, allowed and forbidden regions in the ditopological spaces at their common superpositions), and these correlations/anti-correlations should have a correspondence with, for instance, a discrete spectrum of the volume operator, expected in some quantum gravity models (see, e.g., [22]).

A quantum gravity theory incorporating an underlying concurrent realm should lead to spacetime quanta corresponding to internal competing dynamical actions which avoid the deadlock condition. The macroscopic, continuum limit of such a model should therefore realize spacetime and all dynamical fields purely from concurrency. In this sense, spacetime would be an "homogeneous" reduction (in a Bersonian sense) of the underlying heterogeneous nature of the concurrent substrate. From such a reduction, classical spacetime – most notably, time – should emerge.

What we understand as a particle following its spacetime geodesic would correspond to an underlying trace of concurrent processes that avoids paths leading to "forbidden regions", and consequently to deadlock states.

V. THE CONCURRENT TIME: NATURE ABHORS DEADLOCKS

Inertial mobility – hence time – emerges because the Universe avoids deadlock conditions. We here agree with a Machian-like view: inertia somehow arises from the Universe at large – not at a physical "large scale" distribution of masses –, but from an underlying concurrent substrate, which also gives rise to quantum entanglement phenomena. The existence of tolopogical "forbidden regions" towards deadlock states is what causes an asymmetry leading to what we observe as irreversibility, be it at a quantum or a classical level. Our surprising conclusion is that, in the fuzzy frontier which lies time, what emerges is a signature *that nature is a huge deadlock avoidance system.* Therefore, to a lonely particle in an otherwise empty universe corresponds an underlying trace of very few possibilities, hence, of a poor and almost homogeneous quality, leading to large "forbidden regions", and therefore, eventual deadlock. Inertia is that underlying competing process character, which for a lonely particle implies poor competing mobility, and hence, no time.

Why does Nature abhor deadlocks?

Evidently, we have no answer. But from a moment's thought, it is a very intriguing and deep change of perspective to consider that nature is an all encompassing intrinsic concurrent body, *with only one law to follow: deadlock avoidance*. There are ways to explore these ideas more formally and consistently[9]. What we have exposed here is evidently of high speculative nature and it would be unwise not to explicitly recognize it as such. But if properly conducted, we believe that such attempts are worthwhile.

On the other hand, it may be quite sensible to just accept that the true nature of time is scientifically meaningless, leaving us with just practical notions that serve us well in certain limits and approximations. Time may be indeed the most unreachable frontier of the metaphysical panorama. Or, in an ironical vein, being the most atemporal, immutable mystery of all − we are doomed to live in time, but the mystery of time is timeless.

[9] The new developments on n-categories are very interesting in this respect (see, e.g., [18]).

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